8.4 Improper Integrals

What you'll Learn About

- How to integrate functions that approach infinity or functions that approach an asymptote

2) $\int_1^\infty \frac{dx}{x^{1/3}} = \int_1^\infty x^{-1/3}\,dx$

$$\lim_{b\to\infty}\int_1^b x^{-1/3}\,dx = \lim_{b\to\infty}\left[\frac{3}{2}x^{2/3}\right]_1^b$$

$$= \lim_{b\to\infty}\left[\left(\frac{3}{2}b^{2/3}\right) - \frac{3}{2}\right] = \infty$$

$$\infty - \frac{3}{2}$$

Area Diverges

6) $\boxed{\int_1^\infty \frac{2dx}{x^3} = 1}$

$$\lim_{b\to\infty}\int_1^b 2x^{-3}\,dx = \lim_{b\to\infty}\left[-x^{-2}\right]_1^b = \lim_{b\to\infty}\left[\frac{-1}{x^2}\right]_1^b$$

$$\lim_{b\to\infty}\left[\frac{-1}{b^2} - (-1)\right] = 1$$

$$0 + 1$$

Area Converges to 1

$$\int_1^\infty \frac{1}{x^{1/3}}$$ Diverged

$$\int_1^\infty \frac{1}{x^1} \longrightarrow \ln x \Big]_1^\infty = \ln(\infty) - \ln(1)$$ Diverges

$$\int_1^\infty \frac{1}{x^3}$$ Converged

$$\int_1^\infty \frac{1}{x^{1/2}}$$ Diverge

$$\int_1^\infty \frac{1}{x^2}$$ converge

	10) $\int_{-\infty}^{0} \frac{dx}{(x-2)^3}$ $\lim_{b\to-\infty} \int_b^0 (x-2)^{-3}dx$ $\lim_{b\to-\infty} \left[-\frac{1}{2}(x-2)^{-2}\right]_b^0$ $\lim_{b\to-\infty} \left[-\frac{1}{2}(-2)^{-2} - \left(-\frac{1}{2}(b-2)^{-2}\right)\right] = \left[-\frac{1}{8} + \frac{1}{2(b-2)^2}\right] = -\frac{1}{8}$
$2 = A(x-1) + B(x-3)$ $x = 1$ $x = 3$ $2 = -2B$ $2 = 2A$ $B = -1$ $1 = A$	14) $\int_{-\infty}^{0} \frac{2dx}{x^2-4x+3} =$ * Use properties before you do limits $\lim_{b\to-\infty} \int_b^0 \frac{1}{(x-3)} + \frac{-1}{(x-1)}$ $\lim_{b\to-\infty} \left[\ln\lvert x-3\rvert - \ln\lvert x-1\rvert\right]_b^0$ $\lim_{b\to-\infty} \left[\ln\left\lvert\frac{x-3}{x-1}\right\rvert\right]_b^0 = \lim_{b\to-\infty}\left[\ln 3 - \ln\left\lvert\frac{b-3}{b-1}\right\rvert\right]$ 1 $\ln 3 - \ln 1$

$\boxed{\ln 3}$

$\lim_{b\to-\infty} b^2e^b = \frac{b^2}{e^b}$

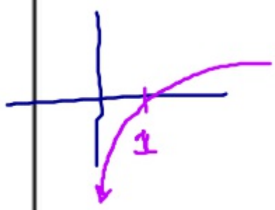
18) $\int_{-\infty}^{0} x^2 e^x \, dx$

x^2	e^x
$2x$	e^x
2	e^x
0	e^x

$$\lim_{b\to-\infty}\int_b^0 x^2e^x\,dx$$

$$\lim_{b\to-\infty}\left[x^2e^x - 2xe^x + 2e^x\right]_b^0$$

$$\lim_{b\to-\infty}\left[2-\left(\underbrace{b^2e^b}_{0} - \underbrace{2be^b}_{0} + \underbrace{2e^b}_{0}\right)\right] = 2$$



43. Find the area of the region in the first quadrant that lies under the given curve

$$y = \frac{\ln x}{x^2}$$

$$\int_1^\infty \frac{\ln x}{x^2}\,dx = \int_1^\infty x^{-2}\ln x\,dx = \int_1 \ln x \; x^{-2}\,dx$$

$\ln x$	x^{-2}
$\frac{1}{x}$	$-x^{-1}$

$$= -\frac{1}{x}\ln x\Big]_1^\infty - \int_1^\infty \frac{1}{x}\cdot\left(-\frac{1}{x}\right)dx$$

$$= -\frac{1}{x}\ln x\Big]_1^\infty + \int_1^\infty \frac{1}{x^2}\,dx$$

$$\lim_{b\to\infty}\left[-\frac{1}{x}\ln x - \frac{1}{x}\right]_1^b$$

$$\lim_{b\to\infty}\left[\left(\cancel{-\frac{1}{b}\ln b} - \frac{1}{b}\right) - \left(0-1\right)\right]$$

$$0 - 0 + 1 = 1$$

$\lim_{b\to\infty} \frac{-\ln b}{b} = \frac{\infty}{\infty}$

L'Hopitals Rule

$\lim_{b\to\infty} \frac{1}{b} = 0$